

# 61 EXPONENTIAL FUNCTIONS ANSWER KEY

**61 EXPONENTIAL FUNCTIONS ANSWER KEY** IS A RESOURCE THAT PROVIDES SOLUTIONS TO A SET OF PROBLEMS INVOLVING EXPONENTIAL FUNCTIONS. THESE FUNCTIONS PLAY A CRITICAL ROLE IN VARIOUS FIELDS SUCH AS MATHEMATICS, PHYSICS, BIOLOGY, AND ECONOMICS. UNDERSTANDING EXPONENTIAL FUNCTIONS IS ESSENTIAL FOR SOLVING REAL-WORLD PROBLEMS THAT INVOLVE GROWTH, DECAY, AND COMPLEX RELATIONSHIPS. IN THIS ARTICLE, WE WILL EXPLORE THE CONCEPT OF EXPONENTIAL FUNCTIONS, THEIR PROPERTIES, AND HOW TO APPROACH PROBLEMS RELATED TO THEM, CULMINATING IN A DETAILED ANSWER KEY.

## UNDERSTANDING EXPONENTIAL FUNCTIONS

EXPONENTIAL FUNCTIONS ARE MATHEMATICAL FUNCTIONS OF THE FORM:

$$f(x) = a \cdot b^x$$

WHERE:

- $a$  IS A CONSTANT THAT REPRESENTS THE INITIAL VALUE,
- $b$  IS THE BASE OF THE EXPONENTIAL FUNCTION (A POSITIVE REAL NUMBER),
- $x$  IS THE EXPONENT OR THE VARIABLE.

THE MOST NOTABLE CHARACTERISTICS OF EXPONENTIAL FUNCTIONS INCLUDE:

## GROWTH AND DECAY

1. **EXPONENTIAL GROWTH:** THIS OCCURS WHEN THE BASE  $(b > 1)$ . THE FUNCTION INCREASES RAPIDLY AS  $(x)$  INCREASES. A COMMON EXAMPLE IS POPULATION GROWTH, WHERE THE POPULATION CAN DOUBLE IN A FIXED AMOUNT OF TIME.

2. **EXPONENTIAL DECAY:** THIS OCCURS WHEN THE BASE  $(0 < b < 1)$ . THE FUNCTION DECREASES RAPIDLY AS  $(x)$  INCREASES. THIS IS OFTEN SEEN IN RADIOACTIVE DECAY OR DEPRECIATION OF ASSETS.

## PROPERTIES OF EXPONENTIAL FUNCTIONS

EXPONENTIAL FUNCTIONS HAVE SEVERAL IMPORTANT PROPERTIES:

- **DOMAIN AND RANGE:** THE DOMAIN IS ALL REAL NUMBERS  $(-\infty, +\infty)$ , WHILE THE RANGE IS RESTRICTED TO POSITIVE REAL NUMBERS  $(0, +\infty)$ .
- **INTERCEPTS:** THE Y-INTERCEPT OCCURS AT  $(0, a)$ , WHILE THERE ARE NO X-INTERCEPTS SINCE THE FUNCTION NEVER TOUCHES THE X-AXIS.
- **ASYMPTOTIC BEHAVIOR:** EXPONENTIAL FUNCTIONS APPROACH ZERO AS  $(x)$  APPROACHES NEGATIVE INFINITY, BUT NEVER ACTUALLY REACH IT.

## SOLVING EXPONENTIAL FUNCTION PROBLEMS

TO SOLVE PROBLEMS INVOLVING EXPONENTIAL FUNCTIONS, FOLLOW THESE STEPS:

1. **IDENTIFY THE TYPE OF PROBLEM:** DETERMINE IF IT INVOLVES GROWTH, DECAY, OR A COMBINATION OF BOTH.
2. **SET UP THE EQUATION:** WRITE DOWN THE FUNCTION BASED ON THE INFORMATION PROVIDED.
3. **SOLVE FOR THE VARIABLE:** THIS MAY INVOLVE TAKING LOGARITHMS OR REARRANGING THE FUNCTION.
4. **INTERPRET THE RESULTS:** ENSURE THAT THE SOLUTION MAKES SENSE IN THE CONTEXT OF THE PROBLEM.

## COMMON TECHNIQUES

- GRAPHING: VISUAL REPRESENTATIONS CAN HELP UNDERSTAND THE BEHAVIOR OF EXPONENTIAL FUNCTIONS.
- USING LOGARITHMS: SINCE EXPONENTIAL FUNCTIONS CAN BE INVERTED USING LOGARITHMS, THIS IS A VITAL TOOL FOR SOLVING EQUATIONS.
- EXPONENTIAL EQUATIONS: THESE OFTEN TAKE THE FORM  $(a \cdot b^x = c)$ . YOU CAN SOLVE FOR  $(x)$  BY REWRITING THE EQUATION IN LOGARITHMIC FORM.

## SAMPLE PROBLEMS AND SOLUTIONS

NOW LET'S LOOK AT SOME SAMPLE PROBLEMS RELATED TO EXPONENTIAL FUNCTIONS, ALONG WITH THEIR SOLUTIONS. THIS SECTION WILL PROVIDE VALUABLE INSIGHTS AND STRENGTHEN YOUR UNDERSTANDING.

### PROBLEM 1: EXPONENTIAL GROWTH

A POPULATION OF RABBITS DOUBLES EVERY YEAR. IF THE INITIAL POPULATION IS 100 RABBITS, HOW MANY RABBITS WILL THERE BE AFTER 5 YEARS?

SOLUTION:

1. IDENTIFY THE FUNCTION: THE EXPONENTIAL GROWTH FUNCTION IS GIVEN BY  $(P(t) = P_0 \cdot 2^t)$ , WHERE  $(P_0 = 100)$ .
2. SET UP THE EQUATION:  
$$P(5) = 100 \cdot 2^5$$
3. CALCULATE:  
$$P(5) = 100 \cdot 32 = 3200$$
4. INTERPRET THE RESULT: AFTER 5 YEARS, THERE WILL BE 3200 RABBITS.

### PROBLEM 2: EXPONENTIAL DECAY

A RADIOACTIVE SUBSTANCE HAS A HALF-LIFE OF 3 YEARS. IF YOU START WITH 80 GRAMS, HOW MUCH WILL REMAIN AFTER 9 YEARS?

SOLUTION:

1. IDENTIFY THE FUNCTION: THE DECAY FUNCTION CAN BE MODELED AS  $(N(t) = N_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}})$ , WHERE  $(T_{1/2} = 3)$  YEARS.
2. SET UP THE EQUATION:  
$$N(9) = 80 \cdot \left(\frac{1}{2}\right)^{\frac{9}{3}} = 80 \cdot \left(\frac{1}{2}\right)^3$$
3. CALCULATE:  
$$N(9) = 80 \cdot \frac{1}{8} = 10$$
4. INTERPRET THE RESULT: AFTER 9 YEARS, 10 GRAMS OF THE SUBSTANCE WILL REMAIN.

## PROBLEM 3: SOLVING EXPONENTIAL EQUATIONS

SOLVE FOR  $x$  IN THE EQUATION  $3 \cdot 2^x = 24$ .

SOLUTION:

1. SET UP THE EQUATION:

$$2^x = \frac{24}{3} = 8$$

2. REWRITE IN LOGARITHMIC FORM:

$$2^x = 2^3$$

3. EQUATE THE EXPONENTS:

$$x = 3$$

4. INTERPRET THE RESULT: THE SOLUTION TO THE EQUATION IS  $x = 3$ .

## 61 EXPONENTIAL FUNCTIONS ANSWER KEY

TO ASSIST WITH THE UNDERSTANDING OF EXPONENTIAL FUNCTIONS, WE PROVIDE A COMPREHENSIVE ANSWER KEY TO A SET OF COMMON PROBLEMS INVOLVING THESE FUNCTIONS. BELOW IS A LIST OF PROBLEMS AND THEIR CORRESPONDING SOLUTIONS.

1. PROBLEM:  $f(x) = 5 \cdot 3^x$ , FIND  $f(2)$ .  
ANSWER:  $f(2) = 45$

2. PROBLEM:  $N(t) = 50 \cdot \left(\frac{1}{2}\right)^t$ , FIND  $N(4)$ .  
ANSWER:  $N(4) = 3.125$

3. PROBLEM: SOLVE  $4^x = 64$ .  
ANSWER:  $x = 3$

4. PROBLEM: FIND  $a$  IF  $2^a = 32$ .  
ANSWER:  $a = 5$

5. PROBLEM: A TREE GROWS AT A RATE OF 10% PER YEAR. IF IT IS CURRENTLY 200 CM TALL, HOW TALL WILL IT BE AFTER 3 YEARS?  
ANSWER: 266.2 CM

## CONCLUSION

EXPONENTIAL FUNCTIONS ARE FUNDAMENTAL IN MODELING A WIDE RANGE OF PHENOMENA IN THE NATURAL AND SOCIAL SCIENCES. BY UNDERSTANDING THEIR PROPERTIES AND HOW TO SOLVE RELATED PROBLEMS, LEARNERS CAN EFFECTIVELY TACKLE VARIOUS CHALLENGES THAT INVOLVE GROWTH AND DECAY. THE 61 EXPONENTIAL FUNCTIONS ANSWER KEY NOT ONLY SERVES AS A PRACTICAL GUIDE BUT ALSO AS A VALUABLE LEARNING TOOL TO ENHANCE PROBLEM-SOLVING SKILLS IN MATHEMATICS AND BEYOND.

# FREQUENTLY ASKED QUESTIONS

## WHAT IS AN EXPONENTIAL FUNCTION?

AN EXPONENTIAL FUNCTION IS A MATHEMATICAL FUNCTION OF THE FORM  $f(x) = a b^x$ , WHERE 'A' IS A CONSTANT, 'B' IS THE BASE OF THE EXPONENTIAL ( $b > 0$  AND  $b \neq 1$ ), AND 'X' IS THE EXPONENT.

## HOW DO YOU IDENTIFY THE BASE IN AN EXPONENTIAL FUNCTION?

THE BASE 'B' IN AN EXPONENTIAL FUNCTION  $f(x) = a b^x$  IS THE NUMBER THAT IS RAISED TO THE POWER OF 'X'. IT DETERMINES THE GROWTH OR DECAY RATE OF THE FUNCTION.

## WHAT IS THE SIGNIFICANCE OF THE CONSTANT 'A' IN THE FUNCTION $f(x) = a b^x$ ?

THE CONSTANT 'A' REPRESENTS THE INITIAL VALUE OR THE Y-INTERCEPT OF THE EXPONENTIAL FUNCTION WHEN  $x = 0$ , GIVING INSIGHT INTO THE STARTING POINT OF THE FUNCTION.

## WHAT ARE THE CHARACTERISTICS OF THE GRAPH OF AN EXPONENTIAL FUNCTION?

THE GRAPH OF AN EXPONENTIAL FUNCTION IS CONTINUOUS AND SMOOTH, HAS A HORIZONTAL ASYMPTOTE (USUALLY THE X-AXIS), AND EITHER INCREASES RAPIDLY (IF  $b > 1$ ) OR DECREASES RAPIDLY (IF  $0 < b < 1$ ).

## HOW CAN EXPONENTIAL FUNCTIONS MODEL REAL-WORLD SCENARIOS?

EXPONENTIAL FUNCTIONS CAN MODEL VARIOUS REAL-WORLD SCENARIOS, SUCH AS POPULATION GROWTH, RADIOACTIVE DECAY, AND COMPOUND INTEREST, WHERE QUANTITIES CHANGE AT RATES PROPORTIONAL TO THEIR CURRENT VALUE.

## WHAT IS THE DOMAIN AND RANGE OF AN EXPONENTIAL FUNCTION?

THE DOMAIN OF AN EXPONENTIAL FUNCTION IS ALL REAL NUMBERS  $(-\infty, \infty)$ , WHILE THE RANGE IS ALWAYS POSITIVE REAL NUMBERS  $(0, \infty)$  FOR THE TYPICAL FORMS OF EXPONENTIAL FUNCTIONS.

## HOW DO YOU FIND THE INVERSE OF AN EXPONENTIAL FUNCTION?

TO FIND THE INVERSE OF AN EXPONENTIAL FUNCTION, YOU SWITCH THE ROLES OF X AND Y, THEN SOLVE FOR Y. THE INVERSE OF  $f(x) = a b^x$  IS  $f^{-1}(x) = \log_b(x/a)$ .

## WHAT IS THE DIFFERENCE BETWEEN EXPONENTIAL GROWTH AND EXPONENTIAL DECAY?

EXPONENTIAL GROWTH OCCURS WHEN THE BASE 'B' IS GREATER THAN 1, LEADING TO AN INCREASE IN VALUE OVER TIME, WHILE EXPONENTIAL DECAY OCCURS WHEN THE BASE 'B' IS BETWEEN 0 AND 1, RESULTING IN A DECREASE IN VALUE.

## HOW DO YOU SOLVE EQUATIONS INVOLVING EXPONENTIAL FUNCTIONS?

TO SOLVE EQUATIONS INVOLVING EXPONENTIAL FUNCTIONS, YOU CAN USE LOGARITHMS TO ISOLATE THE VARIABLE. FOR EXAMPLE, IF YOU HAVE  $b^x = a$ , YOU CAN TAKE THE LOGARITHM BASE B OF BOTH SIDES TO FIND X.

## WHAT ARE SOME COMMON APPLICATIONS OF EXPONENTIAL FUNCTIONS IN TECHNOLOGY?

EXPONENTIAL FUNCTIONS ARE COMMONLY USED IN TECHNOLOGY FOR MODELING DATA GROWTH (LIKE INTERNET USAGE), CALCULATING COMPOUND INTEREST IN FINANCE, AND IN ALGORITHMS RELATED TO COMPUTER SCIENCE AND MACHINE LEARNING.

## **61 Exponential Functions Answer Key**

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